Chapter 4. Moving Charges and Magnetism

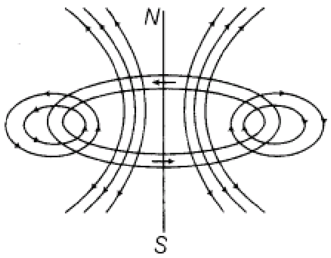
Magnetic Field Laws and their Applications

1 Mark Question

1.Draw the magnetic field lines due to a current carrying loop. [Delhi 2013 C]

Ans.

Magnetic field lines due to a current carrying loop are given by



(1)

2 Marks Questions

2.Considering the case of a parallel plate capacitor being charged, show how one is required

to generalise Ampere's circuital law to include the term due to displacement current. [All India 2014,2011]

Ans.

Ampere's circuital law conduction current during charging of a capacitor was found inconsistent. Therefore, Maxwell modified Ampere's circuital law by introducing displacement current. **(1/2)**

Ampere's circuital law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$ was modified to $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_R + I_D)$

It is called modified Ampere's circuital law or Ampere-Maxwell's circuital law.

The displacement current arising due to time varying electric field is given by $I_D = \varepsilon_0 \frac{d\phi_E}{dt}$

Therefore, modified Ampere's circuital law may be expressed as

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$ **(1/2)**

3.(i) State Biot-Savart's law in vector form expressing the magnetic field due to an element dl carrying current 7 at a distance r from the element.

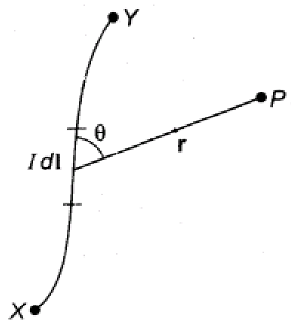
(ii) Write the expression for the magnitude of the magnetic field at the centre of a circular loop of radius r carrying a steady current 7. Draw the field lines due to the current loop.[All India 2014C]

Ans.(i)

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

$dB \propto \sin\theta$

where, θ is the angle between dl and $\mathbf{r}$.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

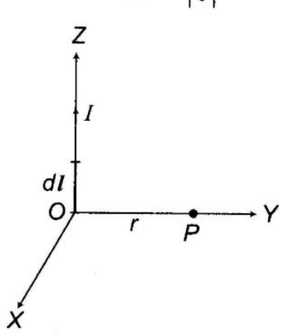
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**

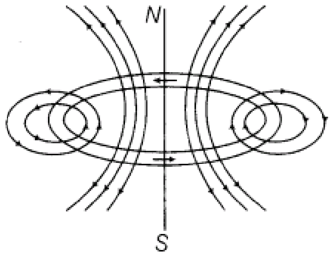


(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

(ii)

Magnetic field lines due to a current carrying loop are given by



(1)

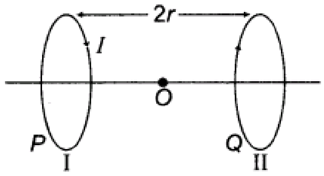
4.Define one tesla using the expression for the magnetic force acting on a particle of charge q moving with velocity v in a magnetic field B. [Foreign 2014]

Ans.

One tesla is defined as the field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of 1m/s.

$$F = qvB \Rightarrow B = \frac{f}{qv} \Rightarrow 1\,\text{T} = \frac{1\,\text{N}}{(1\,\text{C})(1\,\text{m/s})}$$ **(2)**

5.Two identical circular loops P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O.



[Delhi 2012]

Ans.

- To calculate net magnetic field at point O, first of all, calculate the magnetic field at point O due to both coils with direction. By vector addition of these two magnetic fields, net magnetic field can be obtained.

Magnetic field at O due to two rings will be in same direction ($Q \rightarrow P$, along the axis) and of equal magnitude. **(1/2)**

$$B = B_1 + B_2 \text{ but } B_2 = B_1$$

$$\Rightarrow \quad B = 2B_1 = 2\left[\frac{\mu_0 I r^2}{2\,(r^2 + r^2)^{3/2}}\right]$$ **(1/2)**

$$B = \frac{\mu_0 I r^2}{(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2^{3/2} r^3}$$ **(1/2)**

$$B = \frac{\mu_0 I}{2^{3/2} r}$$ **(1/2)**

6.A circular coil of closely wound N turns and radius r carries a current 7. Write the expressions for the following:

(i)The magnetic field at its centre.

(ii)The magnetic moment of this COil. [All India 2012]

Ans.

(*i*) Magnetic field at centre due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$ **(1)**

(*ii*) Magnetic moment, $M = NIA = NI(\pi r^2)$

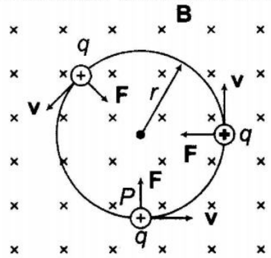
$$M = \pi NIr^2 \quad \textbf{(1)}$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns.

7.A particle of charge q and mass m is moving with velocity It is subjected to a uniform magnetic field B directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius.[Foreign 2012]

Ans.

A charge q projected perpendicular to the uniform magnetic field B with velocity v. The perpendicular force, $F = qvB$, acts like a centripetal force perpendicular to the magnetic field. Then, the path followed by charge is circular as shown in the figure. **(1)**



The Lorentz magnetic force acts as a centripetal force, thus

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

where, $r =$ radius of the circular path followed by charge projected perpendicular to a uniform magnetic field. **(1)**

8. Show how the equation for Ampere's circuital law, *viz*

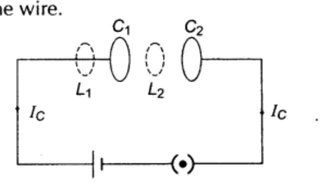
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

is modified in the presence of displacement current.

[Foreign 2011]

Ans.

During charging of a capacitor, let at any instant transient current (I_C) flows through the wire.



Applying Ampere,s circuital law for loops L_1 and L_2, we get

$$\oint_{L_1} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_C$$

$$\oint_{L_2} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times 0 = 0$$

This violates the concept of continuity of electric current. **(1)**

Maxwell introduced the concept of displacement current flowing in space due to varying electric field such that

$$I_C = I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

This maintained continuity of current.

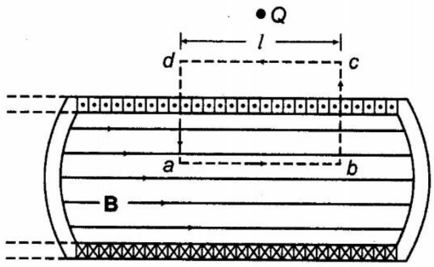
$\therefore$ Modified Ampere's circuital law is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_C + I_D).$$ **(1)**

9.A long solenoid of length L having N turns carries a current I.Deduce the expression for the magnetic field in the interior of the solenoid. [All India 2008; 2011C]

Ans.

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$

(1)

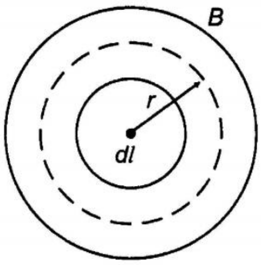
where number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

10.Obtain with the help of a necessary diagram, the expression for the magnetic field in the interior of a toroid carrying current. [HOTS; All India 2011C]

Ans.

Toroid is an endless solenoid to calculate the magnetic field in the interior of toroid, Ampere's circuital law can be obtained.

Toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The direction of the magnetic field at a point is given by tangent to the magnetic field line at that point. **(1)**



$\oint \mathbf{B}\, dl = \int B dl \cos 0° = B\, 2\pi r$ ....(i)

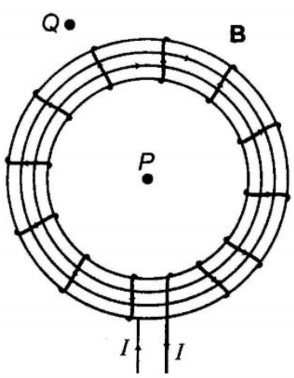
as $\oint \mathbf{B} \cdot dl = \mu_0 I \times$ Number of turns ....(ii)

If n be the number of turns/unit length, then total number of turns $= n \times 2\pi r$

so $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n \times 2\pi r I$ ....(iii)

From Eqs. (i) and (iii), we get

$$B\, 2\pi r = \mu_0 n 2\pi r I$$
$$B = \mu_0 n I$$



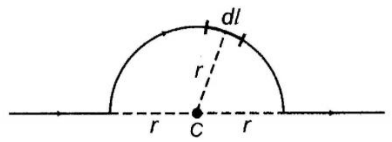
Applying Ampere's circuital law over loop, we have **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ Current passing through the loop

11.A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current 7 passing through it. [Delhi 2011c]

Ans.

. When a straight wire is bent into semi-circular loop, then there are two parts which can produce the magnetic field at the centre one is circular part and other is straight part due to which field is zero.



$\because$ Length L is bent into semi-circular loop.

Length of wire = Circumference of semi-equal circular wire

$$\Rightarrow \quad L = \pi r$$

$$r = \frac{L}{\pi} \quad \ldots(i)$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C.
Using Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{r^2} \quad \textbf{(1/2)}$$

$$[\because Idl \perp \mathbf{r}, \therefore \theta = 90°]$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

$\therefore$ Net magnetic field at C due to semi-circular loop,

$$B = \int_{\text{semicircle}} \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{semicircle}} dl \quad \textbf{(1/2)}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L$$

But, $r = \frac{L}{\pi}$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi} \times \frac{IL}{L^2} \times \pi^2$$

$$\Rightarrow \quad B = \frac{\mu_0 I\pi}{4L} \quad \textbf{(1)}$$

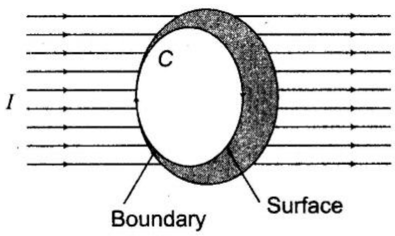
This is required expression.

12.State Ampere's circuital law. Show through an example, how this law enables an easy evaluation of the magnetic field when there is a symmetry in the system? [All India 2010]

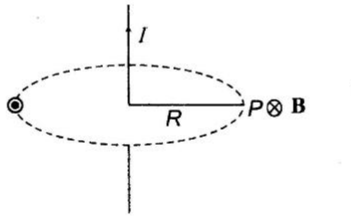
Ans.

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire $\mathbf{B}$ (magnetic field) is tangential to circumference of the loop. **(1)**

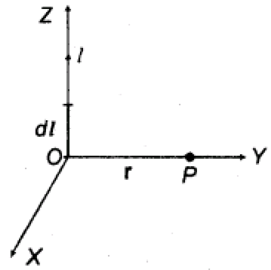
Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\, dl = B\, 2\pi R$

$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

13.State Biot-Savart's law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element dl at a point P situated at a distance r from the element as shown in the figure.[Delhi 2009]



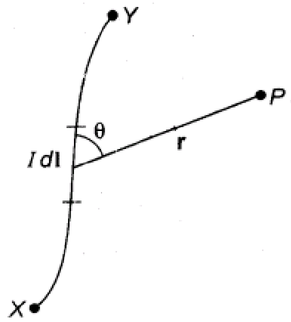
[Delhi 2009]

Ans.

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

$dB \propto \sin\theta$

where, θ is the angle between dl and **r**.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

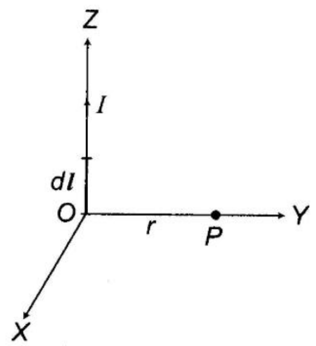
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**



(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

14.A wire of length L is bent round in the form of a coil having N turns of same radius. If a steady current I flows through it in clockwise direction, then find the magnitude and direction of the magnetic field produced at its centre.[Foreign 2009]

Ans.

> When a straight wire is bent in the form of a circular coil of N turns, then the length of the wire is equal to circumference of the coil multiplied by the number of turns.

Let the radius of coil be r.

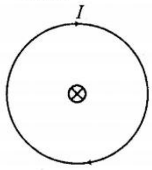
As, the wire is bent round in the form of a coil having N turns.

$\therefore$ $N \times$ circumference of the coil

$= $ Length of the wire

$\therefore \quad (2\pi r) \times N = L$

$$r = \frac{L}{2\pi N} \qquad \text{...(i)}$$



Magnetic field at the centre due to N turns of a coil is given by

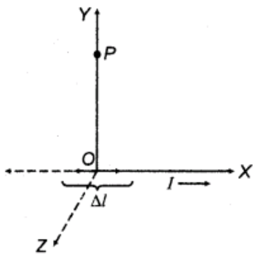
$$B = \frac{\mu_0 (NI)}{2r}$$

$$B = \frac{\mu_0 (NI)}{2\left(\frac{L}{2\pi N}\right)} \qquad \text{[From Eq. (i)]}$$

$$B = \frac{\mu_0 \pi N^2 I}{L}$$ **(1½)**

The direction of magnetic field is perpendicular to the plane of loop and entering into it. **(1/2)**

15. An element $\Delta l = \Delta x I$ is placed at the origin (as shown in figure) and carries a current $I = 2$ A. Find out the magnetic field at a point P on the Y-axis at a distance of 1.0 m due to the element $\Delta x = w$ cm. Also, give the direction of the field produced. **[Delhi 2009C]**



Ans.

Biot-Savart's law states that

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1)**

Here, $\quad I d\mathbf{l} = 2 \times w\hat{\mathbf{i}}$

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{(2w\hat{\mathbf{i}} \times \hat{\mathbf{j}})}{(1)^2}$$ **(1/2)**

$$\hat{\mathbf{r}} = \hat{\mathbf{j}}$$

$$|\mathbf{r}| = 1\text{m}$$

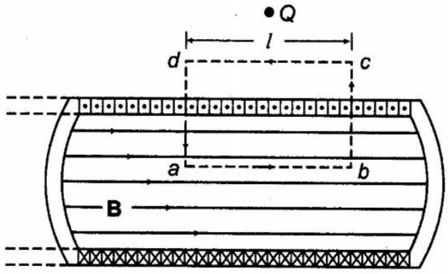
$$d\mathbf{B} = \frac{\mu_0 w}{2\pi}\hat{\mathbf{k}} \Rightarrow |d\mathbf{B}| = \frac{\mu_0 w}{2\pi}$$

and direction along $+Z$-axis. **(1/2)**

16.Using Ampere's circuital law,obtain an expression for the magnetic field along the axis of a current carrying solenoid of length / and having N number of turns. [All India 2008]

Ans.

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

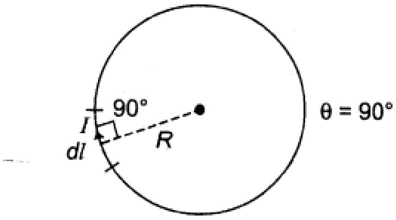
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

17.A circular coil of radius R carries a current /. Write the expression for the magnetic field due to this coil at its centre. Find out the direction Of the field. [All India 2008 C]

Ans.

Consider a small element *dl* on a circular coil of radius *R* carrying current *I*.



$\therefore$ By Biot-Savart's law, magnetic field at the centre due to element of coil.

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{R^2}$$ **(1/2)**

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I}{R^2} dl$$

$\therefore$ Net magnetic field at centre,

$$B = \oint \frac{\mu_0}{4\pi} \frac{I}{R^2} dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \oint dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \times 2\pi R$$

$$= \frac{\mu_0 I}{2R}$$

For N turns of coil, $B = \frac{\mu_0 NI}{2R}$ **(1)**

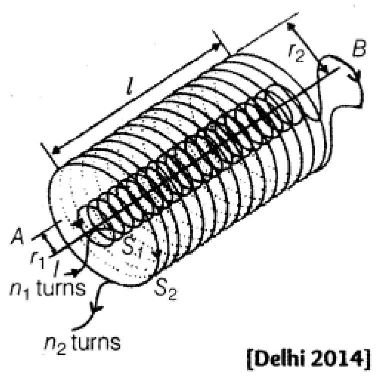
The direction of magnetic field is perpendicular to the plane of paper and directed inwards to the plane. **(1/2)**

3 Marks Questions

18.(i) State Ampere's circuital law expressing it in the integral form, (ii) Two long co-axial insulated solenoids and S_2 of equal length are wound one over the other as shown in the figure.A steady current / flows through the inner solenoid S_x to the other end B which is connected to the outer solenoid through which the some current / flows in the opposite direction so, as to come out at end A. If n_x and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point

(a)inside on the axis and

(b)outside the combined system



[Delhi 2014]

Ans.

(i) Ampere's circuital law states that the line integral of magnetic field (B) around any closed path in vacuum is μ_0 times the net current (I) threading the area enclosed by the curve.

Mathematically, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

Ampere's law is applicable only for an Amperian loop as the Gauss's law is used for gaussian surface in electrostatics. **(1)**

(ii) According to Ampere's circuital law, the net magnetic field is given by $B = \mu_0 n \hat{\mathbf{i}}$ **(1)**

(a) The net magnetic field is given by

$$B_{\text{net}} = B_2 - B_1$$
$$= \mu_0 n_2 I_2 - \mu_0 n_1 I_1 \quad [\because I_2 = I_1 = I]$$
$$= \mu_0 I (n_2 - n_1)$$

The direction is from B to A. **(1)**

(b) As the magnetic field due to S_1 is confined solely inside S_1 as the solenoids are assumed to be very long. So, there is no magnetic field outside S_1 due to current in S_1, similarly there is no field outside S_2.

$\therefore \quad B_{\text{net}} = 0$ **(1)**

19.(i) How is a toroid different from a solenoid?

(ii) Use Ampere's circuital law to obtain the magnetic field inside a toroid.

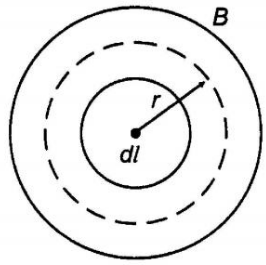
(iii)Show that in an ideal toroid the [1] magnetic field (a) inside the toroid and (b) outside the toroid at any point in the open space is zero. [All India 2014 C]

Ans.(i) A toroid can be viewed as a solenoid which has been bent into circular shape to close on itself.

(ii)

- ? Toroid is an endless solenoid to calculate the magnetic field in the interior of toroid, Ampere's circuital law can be obtained.

Toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The direction of the magnetic field at a point is given by tangent to the magnetic field line at that point. **(1)**



$$\oint \mathbf{B}\, dl = \int B dl \cos 0° = B\, 2\pi r \quad \text{....(i)}$$

as $\oint \mathbf{B} \cdot dl = \mu_0 I \times$ Number of turns ....(ii)

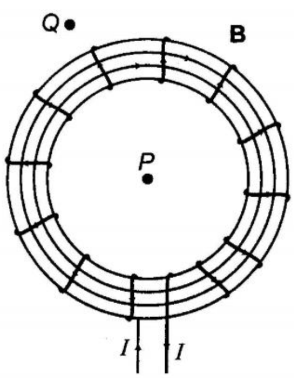
If n be the number of turns/unit length, then total number of turns $= n \times 2\pi r$

so $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n \times 2\pi r I$ ....(iii)

From Eqs. (i) and (iii), we get

$$B\, 2\pi r = \mu_0 n 2\pi r I$$

$$B = \mu_0 n I$$

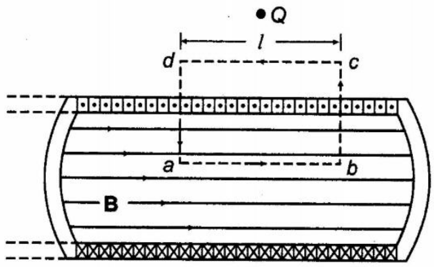


Applying Ampere's circuital law over loop, we have **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ Current passing through the loop

(iii)For the evaluation of magnetic field for a symmetrical system, we can consider the example of a current carrying solenoid. Now,

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

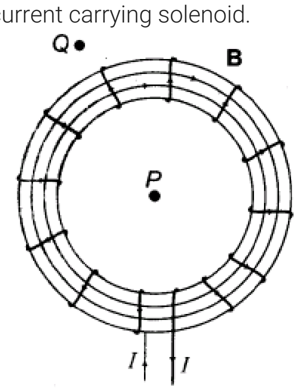
$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

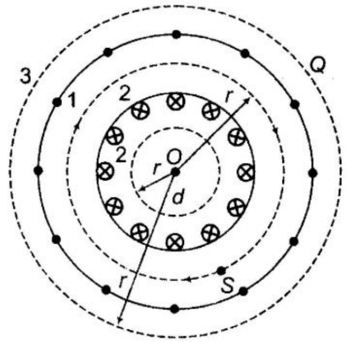
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \qquad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.



(a) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI)$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop. Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 NI$$

$$B \oint_{\text{loop}} dl = \mu_0 NI$$

$$B \times 2\pi r = \mu_0 NI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r}$$ **(1)**

(b) **Magnetic field inside the open space interior the toroid.** Let the loop (2) is shown in figure experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ Ampere's circuital law

$$\oint_{\text{loop 2}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (\theta) = 0 \Rightarrow B = 0$$ **(1)**

Magnetic field in the open space exterior of toroid Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

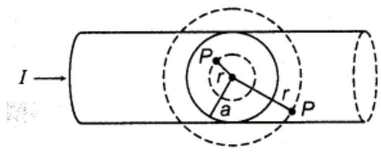
$$= NI - NI = 0$$

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI - NI) = 0 \Rightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior of toroid. **(1)**

20. Figure shows a long straight wire of a circular cross-section of radius a carrying steady current I. The current I is uniformly distributed across this cross-section. Derive the expressions for the magnetic field in the region $r < a$ and $r > a$.

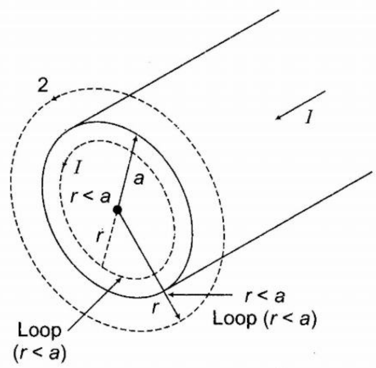


[All India 2011C]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= \dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i) (1/2)}$$

(*i*) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \text{(1/2)}$$

$$\oint Bdl \cos 0^\circ = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2} \quad \text{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

(*ii*) Considering a loop of radius r whose centre lies at the axis of wire and $(r > a)$ as shown in outer dotted line.

$\therefore$ Current I threads the loops.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$ **(1/2)**

$$\oint Bdl \cos 0° = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ **(1)**

$$\Rightarrow \qquad B \propto \frac{1}{r}$$

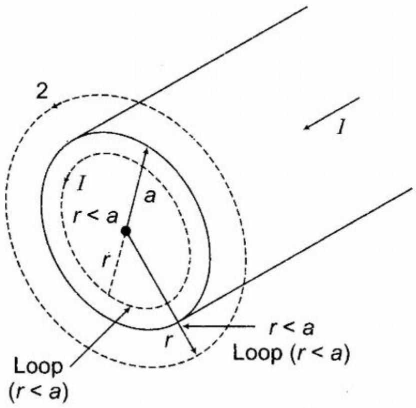
21. A long straight wire of a circular cross-section of radius a carries a steady current I. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point in the region for (*i*) $r < a$ and (*ii*) $r > a$.

[Delhi 2010]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= \dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i)} \quad \textbf{(1/2)}$$

(*i*) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \textbf{(1/2)}$$

$$\oint Bdl \cos 0^\circ = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 I r^2}{a^2} \Rightarrow B = \frac{\mu_0 I r}{2\pi a^2} \quad \textbf{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

(*ii*) Considering a loop of radius r whose centre lies at the axis of wire and ($r > a$) as shown in outer dotted line.

$\therefore$ Current I threads the loops.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \textbf{(1/2)}$$

$$\oint Bdl \cos 0^\circ = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \textbf{(1)}$$

$$\Rightarrow \quad B \propto \frac{1}{r}$$

22.A solenoid of length 1.0 m has a radius of 1 cm and has a total of 1000 turns wound on it. It carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 10^4 m/s along the axis of this current carrying solenoid, what would be the force experienced by this electron? [Delhi 2008 C]

Ans.

- We have to calculate the axial magnetic field inside the solenoid. In a solenoid, the magnetic field is along its axis, so it is called axial magnetic field. So, to find the axial magnetic field inside the solenoid, its regular formula will be used.

Given, $L = 1\text{m}$, $r = 1\text{cm} = 0.01$ m,

$N = 1000$, $I = 5$ A

$\therefore$ Magnetic field P inside the solenoid

$$B = \mu_0 nI \quad \textbf{(1/2)}$$

$$= \mu_0 \left(\frac{N}{L}\right) I$$

$$= \mu_0 \left(\frac{1000}{1}\right) \times 5$$

$$= 4\pi \times 10^{-7} \times 1000 \times 5$$

$$B = 2\pi \times 10^{-3}\ \text{T} \quad \textbf{(1½)}$$

The direction of B is along the axis of solenoid.

Now, $q = -e$,

$v = 10^4$ m/s

and the angle between B and v is $0°$

($\because$ electron moves along the direction of the magnetic field)

$\therefore$ Magnetic Lorentz force,

$$F_B = qvB \sin 0°$$

$$= qvB \times 0 = 0$$

$$F_B = 0$$

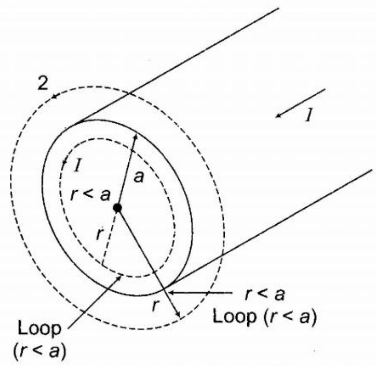
$\Rightarrow$ No magnetic force experienced by the electron. **(1)**

23.A long straight wire of a circular cross-section of radius a carries a steady current /. The current is uniformly distributed across the cross-section of the wire. Use Ampere's circuital law to show that the magnetic field due to this wire in the region inside the wire increases in direct proportion to the distance of the field point from the axis of the wire. Write the value of this magnetic field on the surface of the wire.[All India 2008 C]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= \dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i) (1/2)}$$

(i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \textbf{(1/2)}$$

$$\oint B dl \cos 0^\circ = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2} \quad \textbf{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

$$\because \quad B = \frac{\mu_0 I}{2\pi a^2} r$$

$$\Rightarrow \quad B \propto r \quad \textbf{(1/2)}$$

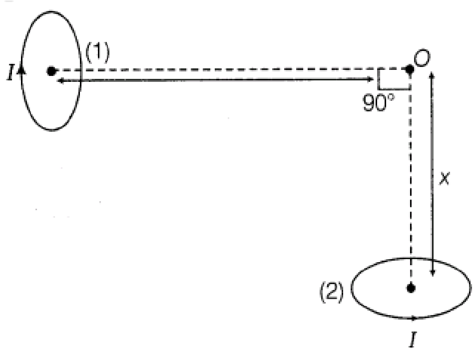
Now, the value of magnetic field on the surface of wire, i.e.

$$r = a$$

$$B = \frac{\mu_0 I}{2\pi a^2} \times a = \frac{\mu_0 I}{2\pi a} \quad \textbf{(1/2)}$$

5 Marks Questions

24.Two very small identical circular loop(1) and (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O.[Delhi 2014]



Ans.

The magnetic field at a point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$$ **(1)**

where,

I = current through the loop

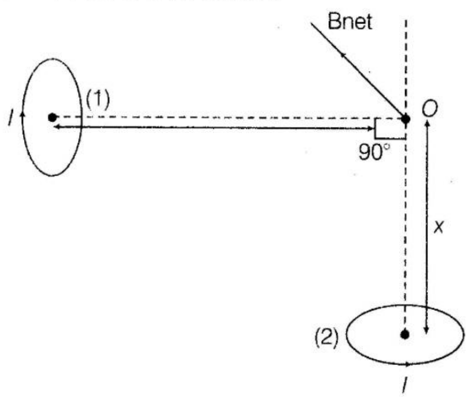
a = radius of the loop

r = distance of O from the centre of the loop.

Since I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by **(1)**

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown below: **(1)**



(1)

The magnitude of the net magnetic field is given by

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$\Rightarrow \quad B_{net} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\pi I a^2}{(a^2 + x^2)^{3/2}}$$ **(1)**

25. State Biot-Savart's law expressing it in the vector form. Use it to obtain the expression for the magnetic field at an axial point distance d from the centre of a circular coil of radius a carrying current I. Also, find the ratio of the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $d = a\sqrt{3}$.

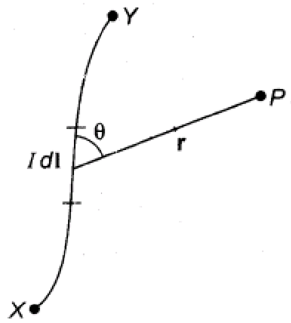
[Delhi 2013C]

Ans..

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

$dB \propto \sin\theta$

where, θ is the angle between dl and $\mathbf{r}$.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

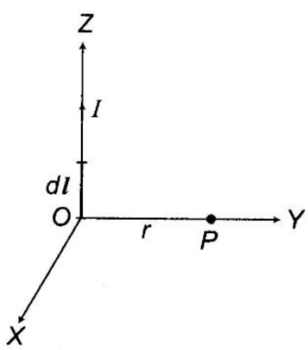
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

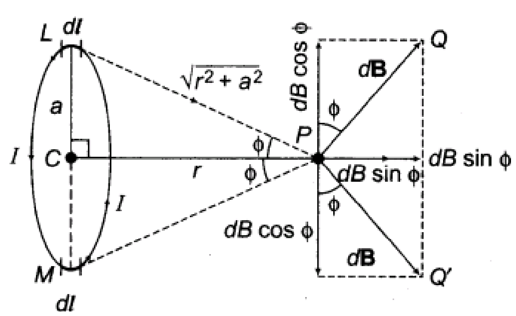
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**



(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

(i)Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element Idl on top L, where current comes out of paper normally, whereas at bottom M enters into the plane paper normally

$\because \quad LP \perp dl$

Also, $\quad MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.
$r =$ distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2}\right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}}\right]$$

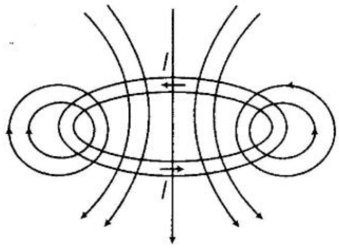
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}}$$

For n turns, $\quad B = \dfrac{\mu_0 n Ia^2}{2\,(r^2 + a^2)^{3/2}}$ **(1)**

The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



In this answer, put $r = d$.

Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a}$$

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 I}{(a^2 + d^2)^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{a^2 \times a}{(a^2 + d^2)^{3/2}} = \frac{a^3}{(a^2 + d^2)^{3/2}}$$

$[\because d = a\sqrt{3}]$

$$\frac{B_1}{B_2} = \frac{a^3}{(a^2 + 3a^2)^{3/2}} = \frac{a^3}{(4a^2)^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{1}{8}$$

(3)

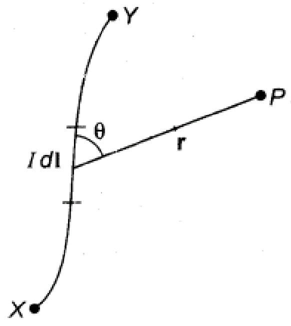
26.State Biot-Savart's law and give the mathematical expression for it. Use law to derive the expression for the magnetic field due to a circular coil carrying current at a point along its axis. How does a circular loop carrying current behave as a magnet?[Delhi 2011]

Ans.For Biot-Savart's law

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(i) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(ii) directly proportional to $\sin\theta$

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where, θ is the angle between dl and $\mathbf{r}$.

(iii) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

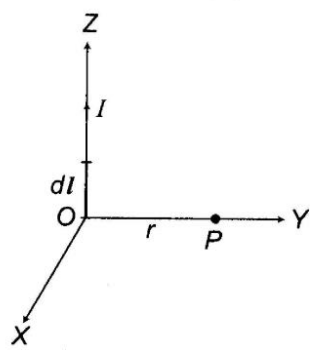
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**

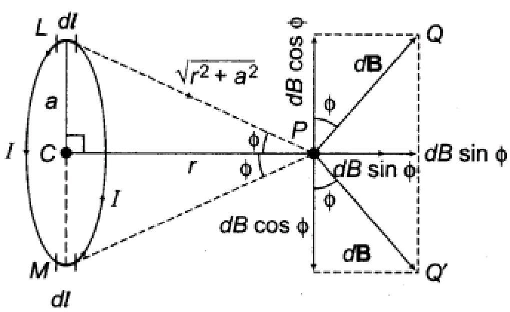


(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

For the magnetic field due to a circular coil carrying current at a point along its axis

(i) Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element **Idl** on top L, where current comes out of paper normally, whereas at bottom **M** enters into the plane paper normally

$\because \qquad LP \perp dl$

Also, $\qquad MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.

r = distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{ In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

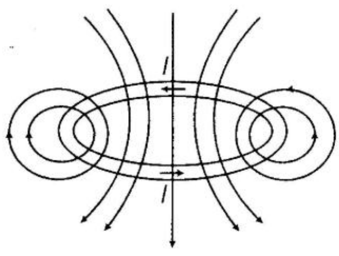
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}}$$

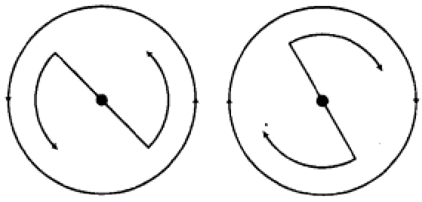
For n turns, $\qquad B = \dfrac{\mu_0 n Ia^2}{2\,(r^2 + a^2)^{3/2}}$ **(1)**

The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



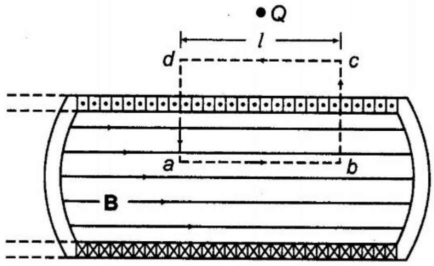
As current carrying loop has the magnetic held lines around it which exists a force on a moving charge. Thus, it behaves as a magnet with two mutually opposite poles



The anti-clockwise flow of current behaves like a North pole, whereas clockwise flow as South pole. Hence, loop behaves as a magnet

27.(i) Using Ampere's circuital law, obtain the expression for the magnetic field due to a long solenoid at a point inside the solenoid on its axis.
(ii) In what respect, is a toroid different from a solenoid? Draw and compare the pattern of the magnetic field lines in the two cases.
(iii) How is the magnetic field inside a given solenoid made strong?[All India 2011]
Ans.(i)

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l =$ length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

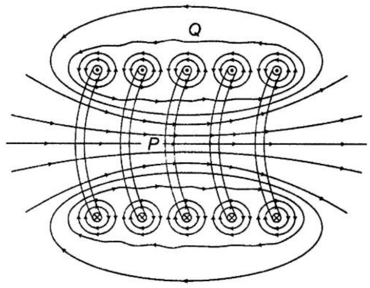
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

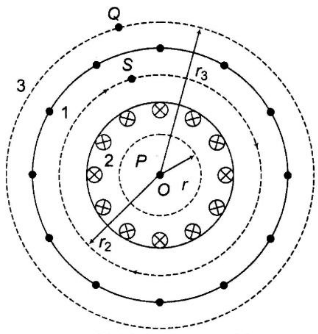
where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

(ii) Solenoid is a hollow circular ring having large number of turns of insulated copper wire on it.
Therefore, we can assume that toroid is a bent solenoid to close on itself.

The magnetic fields due to solenoid and toroid is given in figures below



Field due to solenoid



Field due to toroid

Magnetic field inside the solenoid is a uniform, strong and along its axis also field lines are all most parallel while inside the toroid field line makes closed path.

(iii) The magnetic field in the solenoid can be increased by inserting a soft iron core inside it.

28.(i) State Ampere's circuital law.

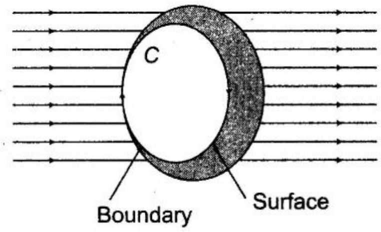
(ii) Use it to derive an expression for magnetic field inside along the axis of an air cored solenoid.

(iii) Sketch the magnetic field lines for a finite solenoid. How are these field lines different from the electric field lines from an electric dipole? [Foreign 2010]

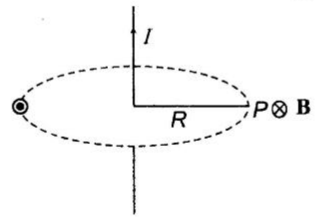
Ans.(i) For statement of Ampere's circuital law

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire $\mathbf{B}$ (magnetic field) is tangential to circumference of the loop. **(1)**

Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\,dl = B\,2\pi R$

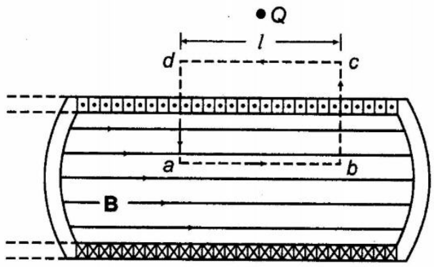
$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

(ii)

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

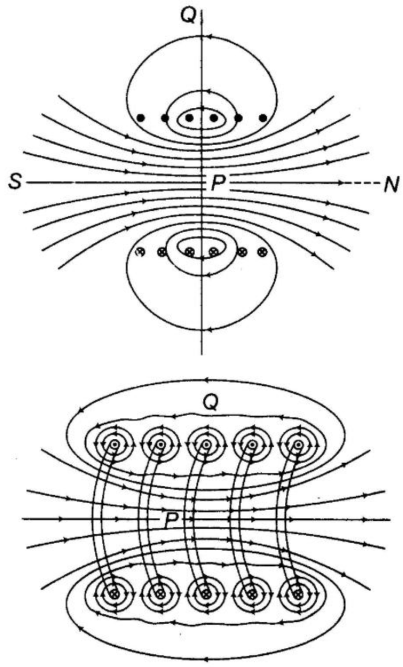
$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

(*iii*) Magnetic field lines due to a finite solenoid has been shown as below:



(2)

All the magnetic field lines are necessarily closed loops, whereas electric lines of force are not.

29.(i)Using Biot-Savart's law, deduce an expression for the magnetic field on the axis of a circular current carrying loop.

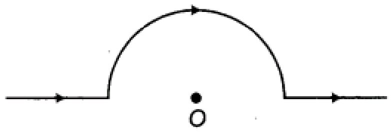
(ii) Draw the magnetic field lines due to a current carrying loop.

(iii) A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in the figure. What is the magnetic field B at O due to

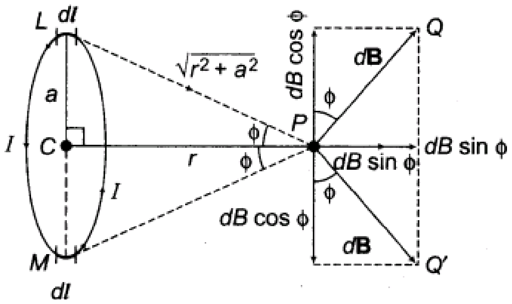
(a)straight segments,

(b)the semi-circular arc?[Foreign 2010]

[Foreign 2010]



Ans.(i) Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element **Idl** on top L, where current comes out of paper normally, whereas at bottom **M** enters into the plane paper normally

$\because \quad LP \perp dl$

Also, $\quad MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.
$r =$ distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2}\right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}}\right]$$

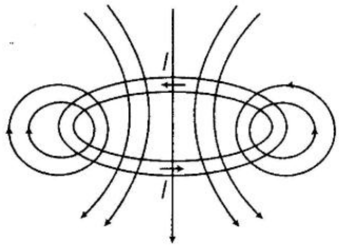
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}}$$

For n turns, $\quad B = \dfrac{\mu_0 n Ia^2}{2\,(r^2 + a^2)^{3/2}}$ **(1)**

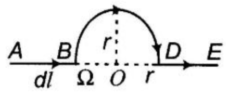
The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



(iii) Magnetic field due to straight part

$$B = \int \frac{\mu_0}{4\pi} \frac{Idl \times \mathbf{r}}{r^3}$$



(1/2)

For point O, dl and r for each element of the straight segments AB and DE are parallel. Therefore, $d\boldsymbol{l} \times \mathbf{r} = 0$. Hence, magnetic field due to straight segments is zero.

Magnetic field at the centre due to circular point

$$= \frac{\text{Magnetic field at the centre of circular coil}}{2}$$

[∵ Here, coil is half]

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4r}$$

$$= \frac{(4\pi \times 10^{-7}) \times 12}{4 \times 2 \times 10^{-2}}$$

$$= 6\pi \times 10^{-5}\ \text{T}$$

(1/2)

30.(i) State Ampere's circuital law. Show through an example, how this law enables an easy evaluation of this magnetic field when there is a symmetry in the system?

(ii) What does a toroid consist of? Show that for an ideal toroid of closely wound turns, the magnetic field.

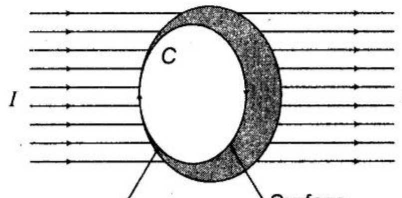
(a)inside the toroid is constant.

(b)in the open space inside an exterior to the toroid is zero.[All India 2010 C]

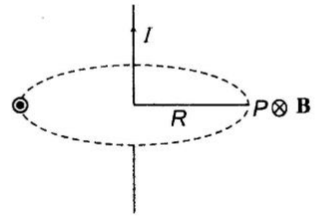
Ans.(i) For statement of Ampere's law

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire $\mathbf{B}$ (magnetic field) is tangential to circumference of the loop. **(1)**

Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\,dl = B\,2\pi R$

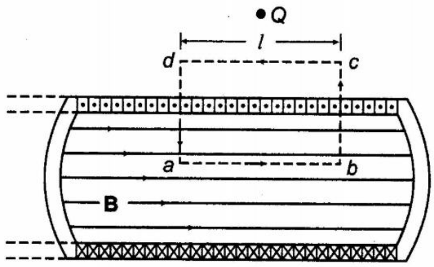
$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

For the evaluation of magnetic field for a symmetrical system, we can consider the example of a current carrying solenoid. Now,

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

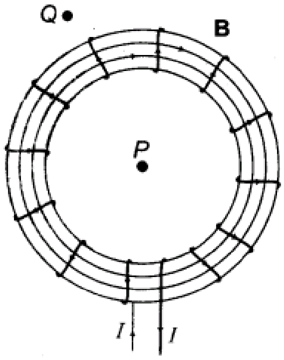
$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

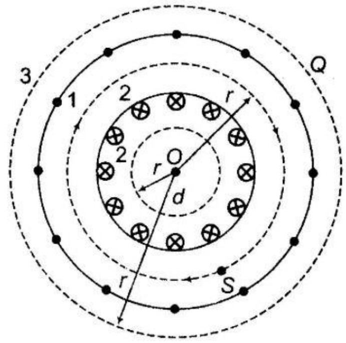
$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$

(1)

where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.



(a) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI)$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop. Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 NI$$

$$B \oint_{\text{loop}} dl = \mu_0 NI$$

$$B \times 2\pi r = \mu_0 NI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r} \qquad \textbf{(1)}$$

(b) **Magnetic field inside the open space interior the toroid.** Let the loop (2) is shown in figure experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ Ampere's circuital law

$$\oint_{\text{loop 2}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (\theta) = 0 \Rightarrow B = 0 \qquad \textbf{(1)}$$

Magnetic field in the open space exterior of toroid Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI - NI) = 0 \Rightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior of toroid. **(1)**

Magnetic Force and Torque

1 Mark Questions

1.Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current. [All India 2014]

Ans.

Force between two straight parallel current carrying conductors

$$F = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r}$$

Let $I_1 = I_2 = 1\,A, r = 1\,m$, then

$$F = 10^{-7} \times \frac{2(1)(1)}{(1)} = 2 \times 10^{-7}$$ **(2)**

One ampere One ampere is that value of current which flows through two straight, parallel infinitely long current carrying conductors placed in air or vacuum at a distance of 1 m and they experience a force of attractive or repulsive nature of magnitude 2×10^{-7} N/m on their unit length. **(1)**

2.Is the steady electric current the only source of magnetic field? Justify your answer.[Delhi 2013 C]

Ans. Yes, the net magnetic force acting on a wire carrying a steady (constant) electric current! in an external magnetic field Band is given F=IdlB

3.Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant? [All India 2008]

Ans.Low torsional constant facilitates greater deflection (allpha) in coil for given value of current and hence, sensitivity of galvanometer increases,

4. Write two factors by which voltage sensitivity of a galvanometer can be increased. [Foreign 2008]

Ans.Voltage sensitivity of galvanometer can be increased by

(i) increasing the magnetic field (ii) decreasing the value of torsional constant,

5. Write two factors by which current sensitivity of a moving coil galvanometer can be increased. [Foreign 2008]

Ans.Current sensitivity of galvanometer can be increased by

(i) increasing the number of turns of coil.

(ii) increasing the magnetic field.

2 Marks Questions

6.A rectangular coil of sider I and b carrying a current I is subjected to a uniform magnetic field B acting perpendicular to its plane. Obtain the expression for the torque acting on it. [Delhi 2014 C]

Ans.

Equivalent magnetic moment of the coil

$$\mathbf{m} = IA\hat{\mathbf{n}}$$

$$\therefore \quad \mathbf{m} = Ilb\,\hat{\mathbf{n}}$$

Wher, $\hat{\mathbf{n}}$ = unit vector $\perp$ to the plane of the coil.

$$\therefore \text{Torque } = \mathbf{m} \times \mathbf{B}$$
$$= Ilb\,\hat{\mathbf{n}} \times \mathbf{B}$$
$$= 0 \qquad \textbf{(1)}$$

As $\hat{\mathbf{n}}$ and $\mathbf{B}$ are parallel or antiparallel to each other.

7.A coil of N turns and radius R carries a current /. It is unwound and rewound to make a square coil of side a having same number of turns N. Keeping the current I same, find the ratio of the magnetic moments of the square coil and the Circular COil. [Delhi 2013 C]

Ans.

Ratio of the magnetic moments

$$\frac{M_s}{M_c} = \frac{2INA_s}{INA_c}$$

$$= \frac{2\left(\frac{R}{2}\right)^2}{(R)^2} = \frac{1}{2}$$

8.A steady current I1 flows through a long straight wire. Another wire carrying steady current I_2 in the same direction is kept close and parallel to the first wire. Show with the help of a diagram, how the magnetic field due to the current I_r exert a magnetic force on the second wire. Deduce the expression for this force. [All India 2011; Foreign 2008]

Ans.

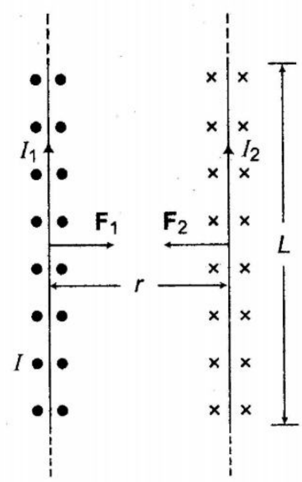
In these types of questions, we are calculating force on a wire in the field produced by the other current carrying wire.

Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction. Magnetic field B_1 due to first wire on seconds, i.e. **(1/2)**

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r} \qquad \text{...(i)}$$

(1/2)

The magnetic field is perpendicular to the plane of paper and directed inwards i.e. (X) type.



Now, magnetic force on L length of second wire is given by **(1/2)**

$$F_2 = I_2 B_1 L \sin 90°$$

$$\Rightarrow \quad F_2 = I_2 \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{r}\right) L$$

$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \text{...(ii)}$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

Similarly, magnetic force on Ist wire is given by

$$\frac{F_1}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2\, I_1 I_2}{r} \quad \text{...(iii)}$$

The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

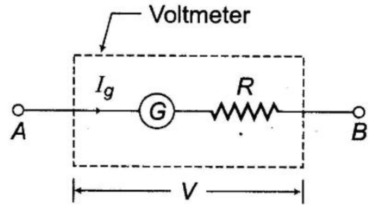
9.How is a moving coil galvanometer converted into a voltmeter? Explain giving the necessary circuit diagram and the required mathematical relation used. [All India 2011 C]

Ans.

> The resistance of an ideal voltmeter is infinity or very high in practical condition. So, to convert a galvanometer into voltmeter, its resistance needs to be increased, which can be done by a high resistance in series connection with it.

A galvanometer can be converted into a voltmeter by connecting a very high resistance R in series with it. **(1/2)**

Let R is so chosen that current I_g gives full deflection in the galvanometer where I_g is the range of galvanometer.



(1/2)

Let galvanometer of resistance G, range I_g is to be converted into voltmeter of range V (volt).

Now,

$$V = I_g (G + R)$$

$$\Rightarrow \quad R + G = \frac{V}{I_g} \Rightarrow R = \frac{V}{I_g} - G$$

The appropriate scale need to be graduated to measure potential difference. **(1)**

10. A square coil of side 10 cm has 20 turns and carries a current of 12 A. The coil is suspended vertically and normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.80 T. If the torque, experienced by the coil equals 0.96 N-m, find the value of θ. **[Delhi 2010C]**

Ans.

Here, Area (A) of coil

$$= 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$$

Number of turns, $N = 20$ turns

Current, $I = 12$ A

Number of the coils make angle with magnetic field $= \theta = ?$

Magnetic field, $B = 0.8$ T

Torque, $\tau = 0.96$ N-m **(1/2)**

$\because$ Torque (τ) experienced by current carrying coil, the magnetic field is

$$\tau = NIAB \sin\theta \quad \textbf{(1)}$$

$$0.96 = 20 \times 12 \times 10^{-2} \times 0.8 \times \sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6} \text{ rad}$$

(1/2)

11.Define current sensitivity and voltage sensitivity of galvanometer. Increasing the current sensitivity may not necessarily increase the voltage sensitivity of a galvanometer, justify your answer. [All India 2009]

Ans.

Current sensitivity The deflection produced in the coil of galvanometer per unit flow of electric current through it, i.e.

Current sensitivity,

$$I_g = \frac{\theta}{I} = \frac{NBA}{K}$$

where abbreviations are as usual. **(1/2)**

Voltage sensitivity The deflection produced in the galvanometer per unit applied potential difference across it.

$\therefore$ Voltage sensitivity

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBA}{KR}$$

where abbreviations are as usual.Its unit is rad|A or div|A **(1/2)**

Increasing the current sensitivity may not necessarily increase the voltage sensitivity, because the current sensitivity increases with the increase of number of turns of the coil but the resistance of coil also increases which affect adversely on voltage sensitivity. **(1)**

3 Marks Questions

12.A metallic rod of length / is rotated with a frequency v with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field B parallel to the axis is present everywhere. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtained the expression for it.[Delhi 2013]

Ans.

Suppose, the length of the rod is greater than the radius of the circle and rod rotates anti-clockwise and suppose, the direction in the rod at any instant be along $+Y$-direction. Suppose, the direction of the magnetic field is along $+Z$-direction.

Then, using Lorentz law, we get the following

$$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$$

$$\Rightarrow \quad \mathbf{F} = -e(v\hat{\mathbf{j}} \times B\hat{\mathbf{k}})$$

$$\Rightarrow \quad \mathbf{F} = -evB\hat{\mathbf{i}} \qquad \textbf{(1)}$$

Thus, the direction of force on the electrons is along X-axis.

Thus, the electrons will move towards the centre, i.e. the fixed end of the rod. This movement of electrons will result in current and hence, it will result in current and hence, it will produce emf in the rod between the fixed end and the point touching the ring.

Let θ be the angle between the rod and radius of the circle at any time t. **(1)**

Then, area swept by the rod inside the circle

$$= \frac{1}{2}\pi r^2\theta$$

Now, induced emf $= B \times \dfrac{d}{dt}\left(\dfrac{1}{2}\pi r^2\theta\right)$

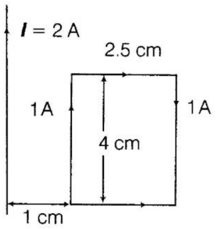
$$= \frac{1}{2}\pi r^2 B\frac{d\theta}{dt} = \frac{1}{2}\pi r^2 B\omega$$

$$= \frac{1}{2}\pi r^2 B(2\pi\nu)$$

$$= \pi^2 r^2 B\nu \qquad \textbf{(1)}$$

NOTE There will be an induced emf between the two ends of the rods also.

13. A rectangular loop of wire of size 2.5 cm × 4 cm carries steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (*i*) torque acting on the loop and (*ii*) the magnitude and direction of the force on the loop due to the current carrying wire. **[Delhi 2012]**



Ans.

There will be force of attraction between the straight wire and 4 cm long arm of loop nearer to the straight conductor.

$$F_1 = \frac{\mu_0}{4\pi}\frac{2 \times 2 \times 1}{(2 \times 10^{-2})} \times (4 \times 10^{-2})$$

[towards straight conductor]

$$F_1 = 8 \times 10^{-7}\,\text{N} \quad \text{...(i)} \quad \textbf{(1)}$$

Similarly, force on other 4 cm arm of loop, away from the straight conductor

$$F_2 = \frac{\mu_0}{4\pi} \times \frac{2 \times 2 \times 1}{(4.5 \times 10^{-2})} \times (4 \times 10^{-2})$$

$$F_2 = 3.55 \times 10^{-7}\,\text{N} \quad \text{...(ii)} \quad \textbf{(1)}$$

[away from conductor]

(*i*) Since, F_1 and F_2 are of different magnitudes, therefore, do not form couple and hence **(1/2)**

Torque, $\tau = 0$

(*ii*) Net force on loop,

$$F = F_1 - F_2$$

[towards straight conductor]

$$F = 8 \times 10^{-7} - 3.55 \times 10^{-7}$$

$$F = 4.45 \times 10^{-7}\,\text{N}$$

The forces on two branches of loop are equal in magnitude and opposite in the directions, hence they balance each other. (1/2)

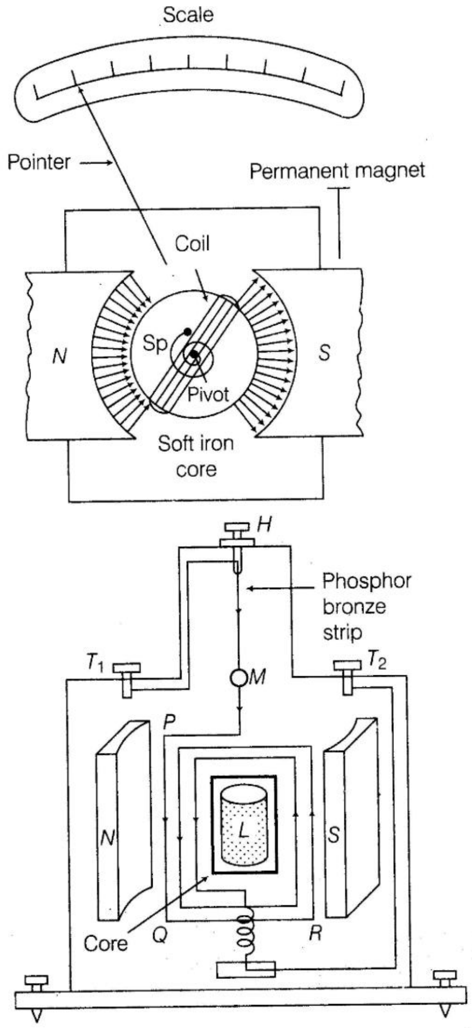
14.Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil?[Foreign 2012]

Ans.

Moving Coil Galvanometer

Principle

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a net torque. **(1)**



(1/2)

Working

Suppose, the coil $PQRS$ is suspended freely in the magnetic field.

Let l = length PQ or RS of the coil

b = breadth QR or SP of the coil

n = number of turns in the coil

Area of each turns of the coil, $A = l \times b$

Let B = strength of the magnetic field in which coil is suspended.

I = current passing through the coil in the direction of $PQRS$

Let at any instant of time, α be the angle which the normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular current carrying coil when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin\alpha$ **(1/2)**

Due to this deflecting torque, the coil rotates and suspended wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and

k be the restoring torque per unit twist of the phosphor bronze strip.

Then,

total restoring torque produced $= k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$\therefore NIBA = k\theta \text{ or } I = \frac{k}{NBA}\theta = G\theta$$

where, $$\frac{k}{NBA} = G$$

[constant for a galvanometer]

It is known as galvanometer constant.

- Current sensitivity of the galvanometer is the deflection per unit current.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

- Voltage sensitivity is the deflection per unit voltage.

$$\therefore \quad \frac{\phi}{V} = \frac{NAB}{k}\left(\frac{I}{V}\right)$$

$$= \frac{NAB}{k}\frac{1}{R} \qquad [\because V = IR]$$

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field, ie. the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil. **(1)**

15.Depict the magnetic field lines due to two straight, long, parallel conductors carrying currents 1\ and 1_2 in the same direction. Hence, deduce an expression for the force per unit length acting on one of the conductors due to the other. Is this force attractive or repulsive? [Delhi 2011 c]

Ans.

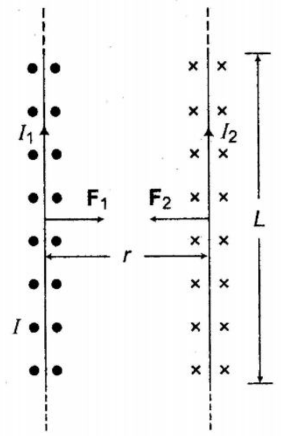
In these types of questions, we are calculating force on a wire in the field produced by the other current carrying wire.

Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction. Magnetic field B_1 due to first wire on seconds, i.e. **(1/2)**

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r} \quad \text{...(i)}$$

(1/2)

The magnetic field is perpendicular to the plane of paper and directed inwards i.e. (X) type.



Now, magnetic force on L length of second wire is given by **(1/2)**

$$F_2 = I_2 B_1 L \sin 90°$$

$$\Rightarrow \quad F_2 = I_2\left(\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r}\right)L$$

$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi}\cdot\frac{2I_1I_2}{r} \quad \text{...(ii)}$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

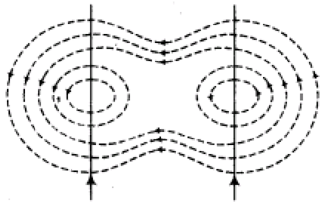
Similarly, magnetic force on Ist wire is given by

$$\frac{F_1}{L} = \frac{\mu_0}{4\pi}\cdot\frac{2\,I_1I_2}{r} \quad \text{...(iii)}$$

The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

(*i*). **(1)**



Magnetic field lines due to both conductors

Current-carrying conductors having same direction of flow of current, so the force between them will be attractive. **(1)**

16.Find the expression for magnetic dipole moment of a revolving electron. What is Bohr magneton?

Ans.

As electric current associated with the revolving electron

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

where, time period $T = \frac{2\pi r}{v}$

r = radius of orbit

v = velocity of electron

The magnetic moment due to the current,

$$\mu = IA = \frac{ev}{2\pi r} \times \pi r^2 \Rightarrow \mu = \frac{evr}{2} \quad \textbf{(1)}$$

If electron revolves in anti-clockwise sense, the current will be in clockwise sense. Hence, according to right hand rule, the direction of magnetic moment must will be perpendicular to the plane of orbit and directed inwards to the plane.

So, $$\mu = \frac{evrm}{2m} = \frac{el}{2m}$$

where, $vrm = l$ = angular momentum orbital of electron and in vector form,

$$\mu = -e\frac{l}{2m}$$

negative sign indicates μ and l are in mutually opposite directions. From Bohr's postulates, $l = mvr = \frac{nh}{2\pi}$, where $n = 1, 2, 3......$

$$\therefore \quad \mu = \frac{e}{2m} \cdot \frac{nh}{2\pi} = n\mu_{min} \quad \textbf{(2)}$$

where, $\mu_{min} = \frac{eh}{4\pi m}$ called Bohr's magneton.

17.State the underlying principle of working of a moving coil galvanometer. Write two reasons why a galvanometer cannot be used as such to measure the current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends. [Delhi 2010]

Ans.

Principle The current carrying coil placed in normal magnetic field experiences a torque which is given by

$$\tau = NIAB$$

where, N = number of turns

I = current

A = area of coil

B = magnetic field **(1)**

The galvanometer cannot be used to measure the current because

(*i*) all the currents to be measured passes through coil and it gets damaged easily as hair line spring or

(*ii*) its coil has considerable resistance because of length and it may affect original current. $\left(\frac{1}{2} \times 2 = 1\right)$

18. A moving coil galvanometer of resistance G gives its full scale deflection when a current I_g flows through its coil. It can be converted into a ammeter of range (0 to I) ($I > I_g$) when a shunt of resistance S is connected is converted into an ammeter of range 0 to 1, find the expression for the shunt required in terms of I_g and G. **[Delhi 2010 C]**

Ans.

> The resistance of an ideal ammeter is zero or very low in practical condition, so to convert a galvanometer into ammeter its resistance needs to be decreased which can be done by connecting a low resistance in its parallel order.

A moving coil galvanometer of range I_g and resistance G can be converted into ammeter by connected very low resistance shunt in parallel with galvanometer.

$\because$ To convert a galvanometer into an ammeter, shunt resistance is connected in parallel with the galvanometer, so the potential difference across the combination is same. **(1)**

$\therefore$ PD across galvanometer = PD across shunt S

$$I_g G = I_s S \quad \textbf{(1)}$$

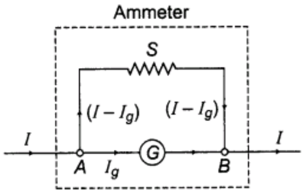
But $I_s + I_g = I$

$\Rightarrow \quad I_s = I - I_g$

$\Rightarrow \quad I_g G = (I - I_g) S$

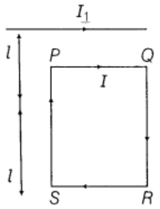
$$\Rightarrow \quad S = \frac{I_g G}{I - I_g} \quad \textbf{(1)}$$

The shunt resistance S to be connected to convert galvanometer into an ammeter.



19.Write the expression for the magnetic moment (m) due to a planar square loop of side / carrying a steady current / in a vector form. In the given figure, this loop is placed in a horizontal plane near a long straight conductor carrying a steady current at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the

expression for this force acting on the loop. **[Delhi 2010]**



Ans.

The magnetic moment of a current carrying loop

$$\mathbf{m} = I\mathbf{A}$$

where, $\mathbf{A}$ = area of the loop (square)

$\therefore \quad \mathbf{A} = l^2\hat{\mathbf{n}}$

Here, $\hat{\mathbf{n}}$ is a unit vector normal to the direction of area vector.

The forces acting on the arms QR and SP of given (in question figure) loop are equal, mutually opposite and collinear. Hence, they are balanced by one another. **(1)**

Force on arm PQ, $F_1 = B_1 Il = \frac{\mu_0 I_1}{2\pi l}; \; Il = \frac{\mu_0 I_1 I}{2\pi}$

Obviously, F_1 is of the attractive nature and directed towards MN.

Again, force on arm RS,

$$F_2 = B_2 Il = \frac{\mu_0 I_1 Il}{2\pi(2l)} = \frac{\mu_0 I_1 I}{4\pi} \qquad \textbf{(2)}$$

F_2 is perpendicular to wire RS and directed away from the conductor MN.

$\therefore$ Net force on loop $PQRS$, $\mathbf{F}_{net} = \mathbf{F}_1 + \mathbf{F}_2$

$$\Rightarrow \mathbf{F}_{net} = \mathbf{F}_1 - \mathbf{F}_2 = \frac{\mu_0 I_1 I}{2\pi} - \frac{\mu_0 I_1 I}{4\pi}$$

or $F_{net} = \frac{\mu_0 I_1 I}{4\pi}$ [attractive]

As, $\mathbf{F}_1$ and $\mathbf{F}_2$ are collinear, hence does not produce torque on the loop $PQRS$.

20.Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence, define one ampere. [Delhi 2009]

Ans.

Force between two straight parallel current carrying conductors

$$F = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$$

Let $I_1 = I_2 = 1\,\text{A}$, $r = 1\,\text{m}$, then

$$F = 10^{-7} \times \frac{2(1)(1)}{(1)} = 2 \times 10^{-7} \qquad \textbf{(2)}$$

One ampere One ampere is that value of current which flows through two straight, parallel infinitely long current carrying conductors placed in air or vacuum at a distance of 1 m and they experience a force of attractive or repulsive nature of magnitude 2×10^{-7} N/m on their unit length. **(1)**

21.Deduce the expression for the torque experienced by a rectangular loop carrying a steady current I and place'[4] in a uniform magnetic field B. In dicate the direction of the torque acting on the loop. [Foreign 2009]

Ans.

- Let a current carrying rectangular loop $PQRS$ carrying a steady current I placed in a uniform magnetic field B keeping axis of the coil perpendicular to field as shown in figure. Let at any instant the area vector **A** makes an angle θ with the direction of magnetic field **B**. **(1)**

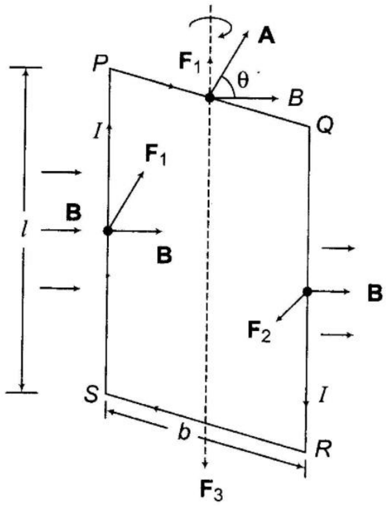


Fig. (a)

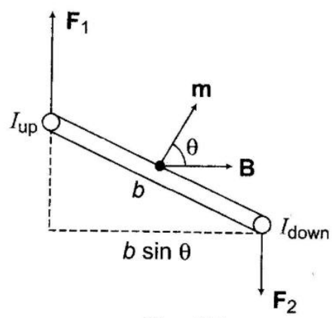


Fig. (b)

Let length and breadth of coil are l and b, respectively.

Now, magnetic force on PS arm of the coil is given by

$$F_1 = IBl \sin 90°$$

$$[\because PS \parallel \text{axis of coil}, \therefore \theta = 90°]$$

$$F_1 = IBl \qquad \text{...(i)}$$

By Fleming's left hand rule, the direction of force is perpendicular to SP and B is along upward direction. Similarly, force of QR arm of the coil.

$$F_2 = IBl \sin 90° \quad \ldots(ii)$$

The direction of force is perpendicular to QR and B is along downward direction.

$\because$ F_1 and F_2 are equal in magnitude, opposite in direction, parallel to each other acting on the loop forms a couple which try to rotate the coil. **(1)**

Now, force on RS part of the coil

$$F_3 = IBb \sin(90° + \theta)$$

$$F_3 = IBb \cos\theta$$

and force on PQ part of the coil

$$F_4 = IBb \sin(90° - \theta) = IBb \cos\theta$$

But Fleming's left hand rule, F_3 and F_4 are equal in magnitude and opposite in direction along the same line of action. Therefore, they balance each other. (cancel out)

Now, torque due to F_1 and F_2 is given by

τ = Force $\times$ perpendicular distance between lines of action of F_1 and F_2.

$$\tau = F \times b \sin\theta$$

But, $F_1 = F_2 = F = IBl$

$$\tau = (IBl) \times (b \sin\theta)$$

$$\tau = IB(lb) \sin\theta$$

$$\tau = IBA \sin\theta$$

where, $A = lb$ = area of coil for N turns of coil

$$\tau = NIAB \sin\theta \quad \textbf{(1)}$$

22. A circular coil of 200 turns and radius 10 cm is placed in a uniform magnetic field of 0.5 T, normal to the plane of the coil. If the current in the coil is 3.0 A, calculate the

(*i*) total torque on the coil

(*ii*) total force on the coil

(*iii*) average force on each electron in the coil due to the magnetic field. Assume the area of cross-section of the wire to be 10^{-5} M^2 and the free electron density is 10^{29} $/M^3$.

[All India 2008]

Ans.

Given, $N = 200, r = 10$ cm $= 0.1$ m, $B = 0.5$ T

NOTE Magnetic field is normal to the plane of coil. Therefore, area vector of coil (which is normal to plane of coil) is along the direction of magnetic field.

$\therefore$ $\theta = 0°$ Also $I = 3$ A

(*i*) As, $\tau = NIAB$

$$= 200 \times 3 \times [\pi (0.1)^2] \times 0.5$$

$[\because A = \pi r^2]$

$\Rightarrow$ $\tau = 9.42$ N-m **(1)**

(*ii*) The net magnetic force on circular loop is zero. **(1/2)**

(iii) $\because$ Average force on electron,

$$F = (-e)(v_d) B \sin 90°$$

But, $I = -ne Av_d$

$$v_d = \frac{I}{-neA}$$

$$\therefore \quad F = (-e)\left(\frac{I}{neA}\right) B$$

$$F = \frac{I_B}{nA} = \frac{3 \times 0.5}{10^{29} \times 10^{-5}}$$

$$F = \frac{1.5}{10^{24}}$$

$$\Rightarrow \quad F = 1.5 \times 10^{-24} \text{ N}$$ **(1½)**

5 Marks Questions

23.(i)Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(ii) Answers the following questions.

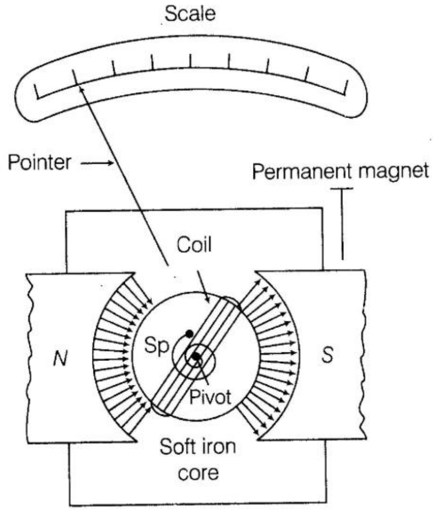
(a) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

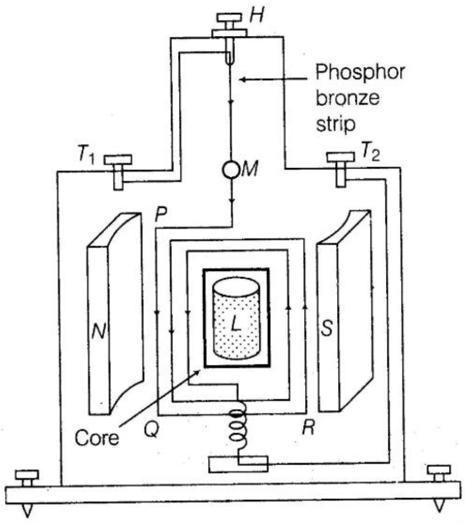
(b)Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain giving reasons. [All India 2014]

Ans.(i)

Moving Coil Galvanometer
Principle

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a net torque. **(1)**





(1/2)

Working

Suppose, the coil $PQRS$ is suspended freely in the magnetic field.

Let l = length PQ or RS of the coil
b = breadth QR or SP of the coil
n = number of turns in the coil

Area of each turns of the coil, $A = l \times b$

Let B = strength of the magnetic field in which coil is suspended.

I = current passing through the coil in the direction of $PQRS$

Let at any instant of time, α be the angle which the normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular current carrying coil when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin\alpha$ **(1/2)**

Due to this deflecting torque, the coil rotates and suspended wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and

k be the restoring torque per unit twist of the phosphor bronze strip.

Then,

total restoring torque produced $= k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$\therefore NIBA = k\theta \text{ or } I = \frac{k}{NBA}\theta = G\theta$$

where, $\frac{k}{NBA} = G$

[constant for a galvanometer]

It is known as galvanometer constant.

- Current sensitivity of the galvanometer is the deflection per unit current.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

- Voltage sensitivity is the deflection per unit voltage.

$$\therefore \quad \frac{\phi}{V} = \frac{NAB}{k}\left(\frac{I}{V}\right)$$

$$= \frac{NAB}{k}\frac{1}{R} \quad [\because V = IR]$$

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field, ie. the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil. **(1)**

(ii) (a) It is necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer because magnetic field is increased, so its sensitivity increases and magnetic field becomes radial. So, angle between the plane of coil and magnetic line of force is zero in all orientations of coil. **(2)**

(b) The relation between current sensitivity and voltage sensitivity is given by

$$\text{Voltage sensitivity} = \frac{\text{Current sensitivity}}{\text{Resistance of coil}}$$

$$\Rightarrow \quad V_S = \frac{I_S}{R_C} \quad \textbf{(1)}$$

If R_C is constant, then $V_S \propto I_S$

This means if V_S increases, I_S also increases.

But, if R_C increases in the given ratio, then V_S is a constant. **(1)**

24.(i) State using a suitable diagram, the working principle of a moving coil galvanometer. What is the function of a radial magnetic field and the soft iron core used in it? (ii) For converting a galvanometer into an ammeter, a shunt resistance of small value is used in parallel, whereas in the case of a voltmeter a resistance of large value is used in series. Explain why? [Delhi 2014C]

Ans.(i)

Principle The current carrying coil placed in normal magnetic field experiences a torque which is given by

$$\tau = NIAB$$

where, N = number of turns

I = current

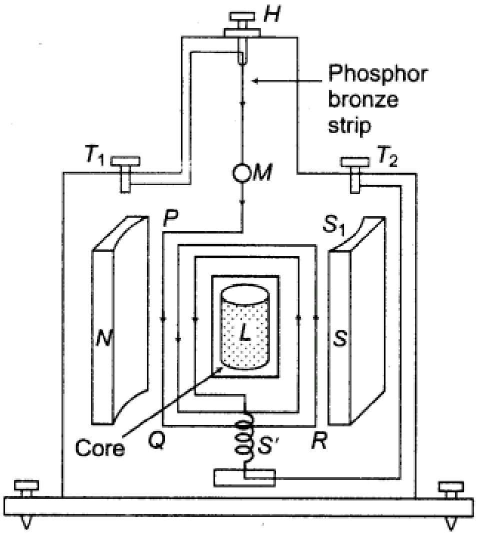
A = area of coil

B = magnetic field **(1)**

The galvanometer cannot be used to measure the current because

(i) all the currents to be measured passes through coil and it gets damaged easily as hair line spring or

(ii) its coil has considerable resistance because of length and it may affect original current. $\left(\frac{1}{2} \times 2 = 1\right)$



(1/2)

The coil remains suspended in radial magnetic field so that it always experiences maximum torque.

When current passes through the coil, deflection torque $\tau(\theta)$ is produced given by

$$\tau_{\text{deflection}} = NIAB \sin 90° \qquad \ldots(i)$$

As a result, coil rotates and phosphor bronze strip gets twisted. As a result restoring torque given by

$$\tau_{\text{restoring}} = k\theta \qquad \ldots(ii)$$

where, k = torsional restoring constant

$\therefore$ In equilibrium,

$$t_{\text{deflecting}} = \tau_{\text{restoring}}$$

$$NIAB = k\theta$$

$$I = \left(\frac{k}{NAB}\right)\theta$$

$$I \propto \theta$$

greater the current, greater the deflection.

(1)

(*ii*) In radial magnetic field, the plane of the coil is always parallel to the plane of the magnetic field and area vector of coil is perpendicular to magnetic field. It is always exerts maximum torque on the coil. **(1)**

(*iii*) The voltmeter connected in parallel with the electrical circuit elements to measure potential difference. For exact measurement of PD voltmeter must draw minimum current which is possible only when it has high resistance. Ammeter is connected in series with the electrical circuit and current to be measured passes through it.

In order to protect the galvanometer, a feeble current must pass through the galvanometer, it is possible only when a low resistance (shunt) is connected in parallel with galvanometer to allow the major part of the current to pass through it. **(2)**

(ii)

(*i*) A galvanometer of range I_g and resistance G_1 can be converted into

(a) a voltmeter of range V by connecting a high resistance R in series with it where value is given by

$$R = \frac{V}{I_s} - G$$

25.(i) Explain giving reasons, the basic difference in converting a galvanometer into (a) a voltmeter and (b) an ammeter.(ii) Two long straight parallel conductors carrying steady currents and I_2 are separated by a distance d. Explain briefly with the help of a suitable diagram, how the magnetic field due to one conductor acts on the other. Hence, deduce the expression for the force acting between the two conductors. Mention the nature of this force.[All India 2012]

Ans.

(*i*) A galvanometer of range I_g and resistance G_1 can be converted into

(a) a voltmeter of range V by connecting a high resistance R in series with it where value is given by

$$R = \frac{V}{I_s} - G$$

(b) an ammeter of range I by connecting a very low resistance (shunt) in parallel with galvanometer whose value is given by

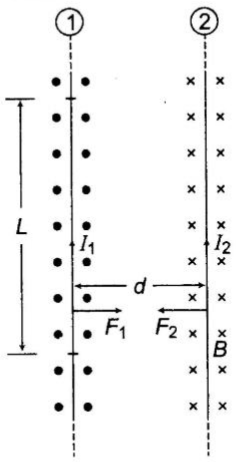
$$S = \frac{I_g G}{I - I_g}$$ **(1)**

(ii) Let two straight wires of infinite length are carrying currents, I_1 and I_2 in the same direction and separated by distance d apart from each other. **(1)**

The magnetic field due to wire 1 at any point on wire 2,

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{d} \quad \text{...(i)}$$

The distance of B_1 is perpendicular to plane of paper and directed inward.



(1/2)

Magnetic force on wire 2 in L length of it

$$F_2 = I_2 B_1 L \sin 90° = I_2 \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{d}\right) L \times 1 \quad \textbf{(1/2)}$$

$$\therefore \quad F_2 = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{d} \quad \text{...(ii)}$$

[towards wire]

By Fleming's left hand rule.

Similarly, force on wire 1 due to wire 2 can be proved $F_1 = \dfrac{\mu_0}{4\pi} \dfrac{2 I_1 I_2}{d}$ ...(iii)

Thus, the nature of force is attractive.

When direction of flow of current gets in opposite direction, the nature of force becomes repulsive. **(2)**

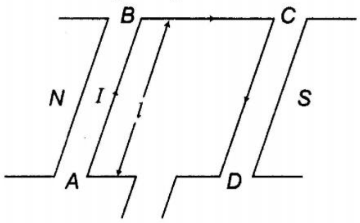
26. A rectangular loop of size $l \times b$ carrying a steady current I is placed in a uniform magnetic field **B**. Prove that the torque τ acting on the loop is given by $\tau = \mathbf{m} \times \mathbf{B}$, where is the magnetic moment of the loop.

[All India 2012]

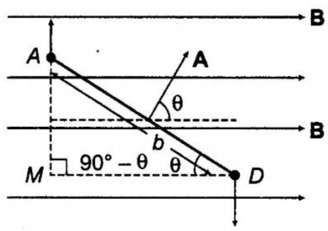
Ans.

Magnetic force on AB and CD parts of wire

$F_1 = F_2 = IBl$ as $\theta = 90°$ **(1/2)**



(1)



(1)

The magnetic force on BC and DA part of wire are equal in magnitude, opposite in direction along the same line. Therefore, they balance each other. **(1/2)**

Let at any instant area vector of coil made an angle θ with the direction of magnetic field.

$\therefore$ F_1 and F_2 form couple which try to rotate the coil.

From figure,

$\therefore$ Torque, τ = force × arm of the couple

$= (IBl) \times MD$

$= IBl \times b \sin\theta = IB\,(lb) \sin\theta$

$\tau = IB\,A \sin\theta$ **(1)**

where, $A = lb$ = area of coil

$\therefore \quad \tau = IAB \sin\theta$

But, $\quad m = IA$

$\therefore \quad \tau = mB \sin\theta$

In vector form,

$\tau = \mathbf{m} \times \mathbf{B}$ **(1)**

27. (*i*) Show that a planer loop carrying a current I, having N closely wound turns and area of cross-section A, possesses a magnetic moment $\mathbf{m} = NI\mathbf{A}$.

(*ii*) When this loop is placed in a magnetic field $\mathbf{B}$, find out the expression for the torque acting on it.

(*iii*) A galvanometer coil of 50 Ω resistance shows full scale deflection for a current of 5 mA. How will you convert this galvanometer into a voltmeter of range 0 to 15 V? **[Foreign 2011]**

Ans.

(*i*) Torque on rectangular loop,

$$\tau = NIAB \sin\theta \quad \text{...(i)}$$

where, symbols are as usual.

Also, torque experienced by magnetic dipole of moment m are placed in a uniform magnetic field.

$$\tau = mB \sin\theta \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

The magnetic dipole moment,

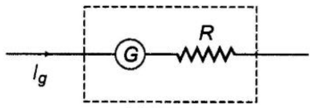
$$m = NIA$$

Also, **m** is along **A**.

$$\Rightarrow \quad \mathbf{m} = NI\mathbf{A} \quad \textbf{(2)}$$

(*ii*) Refer to ans. 21. **(1/2)**

(*iii*) $G = 50\ \Omega,\ I_g = 5 \times 10^{-3}$ A, $V = 15$ V



$$\because \quad V = I_g (G + R)$$

$$\Rightarrow \quad R = \frac{V}{I_g} - G$$

$$= \frac{15}{5 \times 10^{-3}} - 50$$

$$R = 2950\ \Omega \quad \textbf{(1)}$$

A resistance $R = 2950\ \Omega$ is to be connected in series with galvanometer to convert it into a desired voltmeter. **(1½)**

28.(i) With the help of a diagram, explain the principle and working of a moving coil galvanometer.

(ii)What is the importance of radial magnetic field and how is it produced?

(iii)Why is it that while using a moving coil galvanometer as a voltmeter, a high resistance in series is required whereas in an ammeter a shunt is used?[All India 2010; Delhi 2009]

Ans.(i)

Principle The current carrying coil placed in normal magnetic field experiences a torque which is given by

$$\tau = NIAB$$

where, N = number of turns

I = current

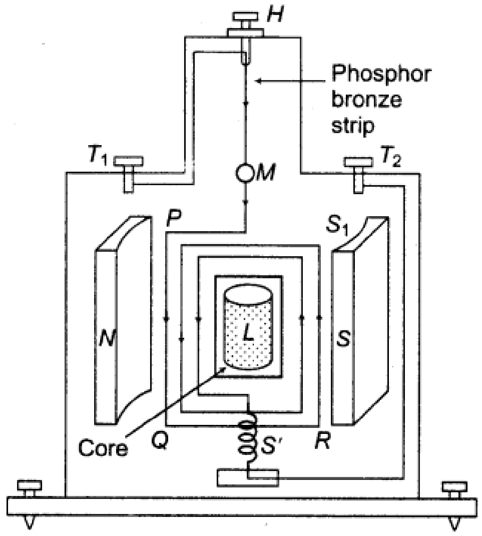
A = area of coil

B = magnetic field **(1)**

The galvanometer cannot be used to measure the current because

(*i*) all the currents to be measured passes through coil and it gets damaged easily as hair line spring or

(*ii*) its coil has considerable resistance because of length and it may affect original current. $\left(\frac{1}{2} \times 2 = 1\right)$



(1/2)

The coil remains suspended in radial magnetic field so that it always experiences maximum torque.

When current passes through the coil, deflection torque $\tau(\theta)$ is produced given by

$$\tau_{deflection} = NIAB \sin 90° \quad \text{...(i)}$$

As a result, coil rotates and phosphor bronze strip gets twisted. As a result restoring torque given by

$$\tau_{restoring} = k\theta \quad \text{...(ii)}$$

where, k = torsional restoring constant

$\therefore$ In equilibrium,

$$t_{deflecting} = \tau_{restoring}$$

$$NIAB = k\theta$$

$$I = \left(\frac{k}{NAB}\right)\theta$$

$$I \propto \theta$$

greater the current, greater the deflection. **(1)**

(*ii*) In radial magnetic field, the plane of the coil is always parallel to the plane of the magnetic field and area vector of coil is perpendicular to magnetic field. It is always exerts maximum torque on the coil. **(1)**

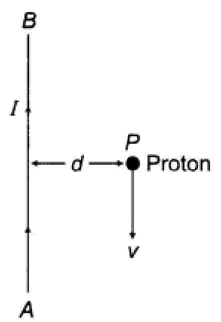
(*iii*) The voltmeter connected in parallel with the electrical circuit elements to measure potential difference. For exact measurement of PD voltmeter must draw minimum current which is possible only when it has high resistance. Ammeter is connected in series with the electrical circuit and current to be measured passes through it.

In order to protect the galvanometer, a feeble current must pass through the galvanometer, it is possible only when a low resistance (shunt) is connected in parallel with galvanometer to allow the major part of the current to pass through it. **(2)**

29.(i) Derive an expression for the force between two long parallel current carrying conductors.

(ii) Use this expression to define SI unit of current.

(iii) A long straight wire AB carries a current /. A proton P travels with a speed v, parallel to the wire at a distance d from it in a direction opposite to the current as shown in the figure. What is the force experienced by the proton and what is its direction?[All India 2010; Foreign 2008]



Ans.(i)

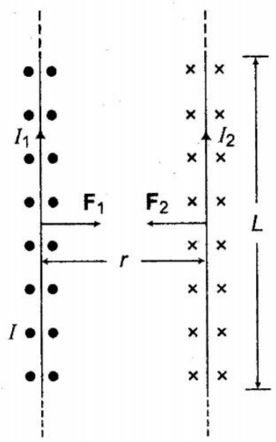
In these types of questions, we are calculating force on a wire in the field produced by the other current carrying wire.

Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction. Magnetic field B_1 due to first wire on seconds, i.e. **(1/2)**

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} \quad \text{...(i)}$$

(1/2)

The magnetic field is perpendicular to the plane of paper and directed inwards i.e. (X) type.



Now, magnetic force on L length of second wire is given by **(1/2)**

$$F_2 = I_2 B_1 L \sin 90°$$

$$\Rightarrow \quad F_2 = I_2 \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{r}\right) L$$

$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \text{...(ii)}$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

Similarly, magnetic force on Ist wire is given by

$$\frac{F_1}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} \quad \text{...(iii)}$$

The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

(ii) As, $\frac{F}{L} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

$I_1 = I_2 = 1\text{ A}$

$\frac{F}{L} = 2 \times 10^{-7}\text{ N/m}$

$r = 1\text{m}$

For definition Refer to ans. 20. **(1)**

(iii) Here, magnetic field due to the current carrying conductor at a distance d from it is given by

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$ **(1/2)**

$\therefore$ Force on proton,

$$F = (e)(v)B\sin 90°$$

$$F = evB$$

$$F = eV\left(\frac{\mu_0}{4\pi}\frac{2I}{d}\right)$$

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2IeV}{d}$$ **(1)**

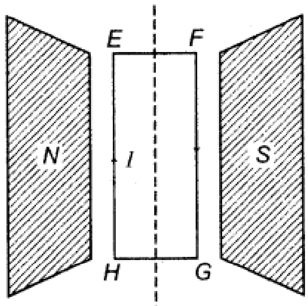
The proton is directed perpendicular to straight conductor and away from it. **(1/2)**

30.(i) Two straight long parallel conductors carry currents $!_x$ and I_2 in the same direction. Deduce the expression for the force per unit length between them. Depict the pattern of magnetic field lines around them.

(ii) A rectangular current carrying loop EFGH is kept in a uniform magnetic field as shown in the figure.

(a) What is the direction of the magnetic moment of the current loop?

(b) What is the torque acting on the loop maximum and zero?



[Foreign 2010; Delhi 2009]

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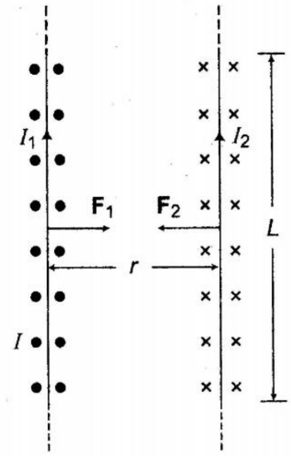
Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction. Magnetic field B_1 due to first wire on seconds, i.e. **(1/2)**

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} \quad \text{...(i)}$$

(1/2)

The magnetic field is perpendicular to the plane of paper and directed inwards i.e. (X) type.



Now, magnetic force on L length of second wire is given by **(1/2)**

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$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \text{...(ii)}$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

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The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

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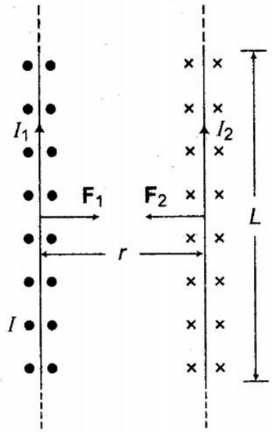

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$$\Rightarrow \quad \frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \text{...(ii)}$$

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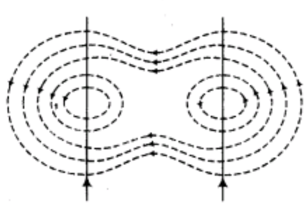
Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other. **(1/2)**

(*i*). **(1)**



Magnetic field lines due to both conductors

Current-carrying conductors having same direction of flow of current, so the force between them will be attractive. **(1)**

(*ii*) (*a*) Perpendicular to the plane of the paper and directed inward. **(1)**

(*b*) When angle between area vector of coil and magnetic field is 90°, then maximum torque experienced by the coil.

When $\theta = 0°$ or $180°$, then torque will be minimum, i.e. zero. **(1)**